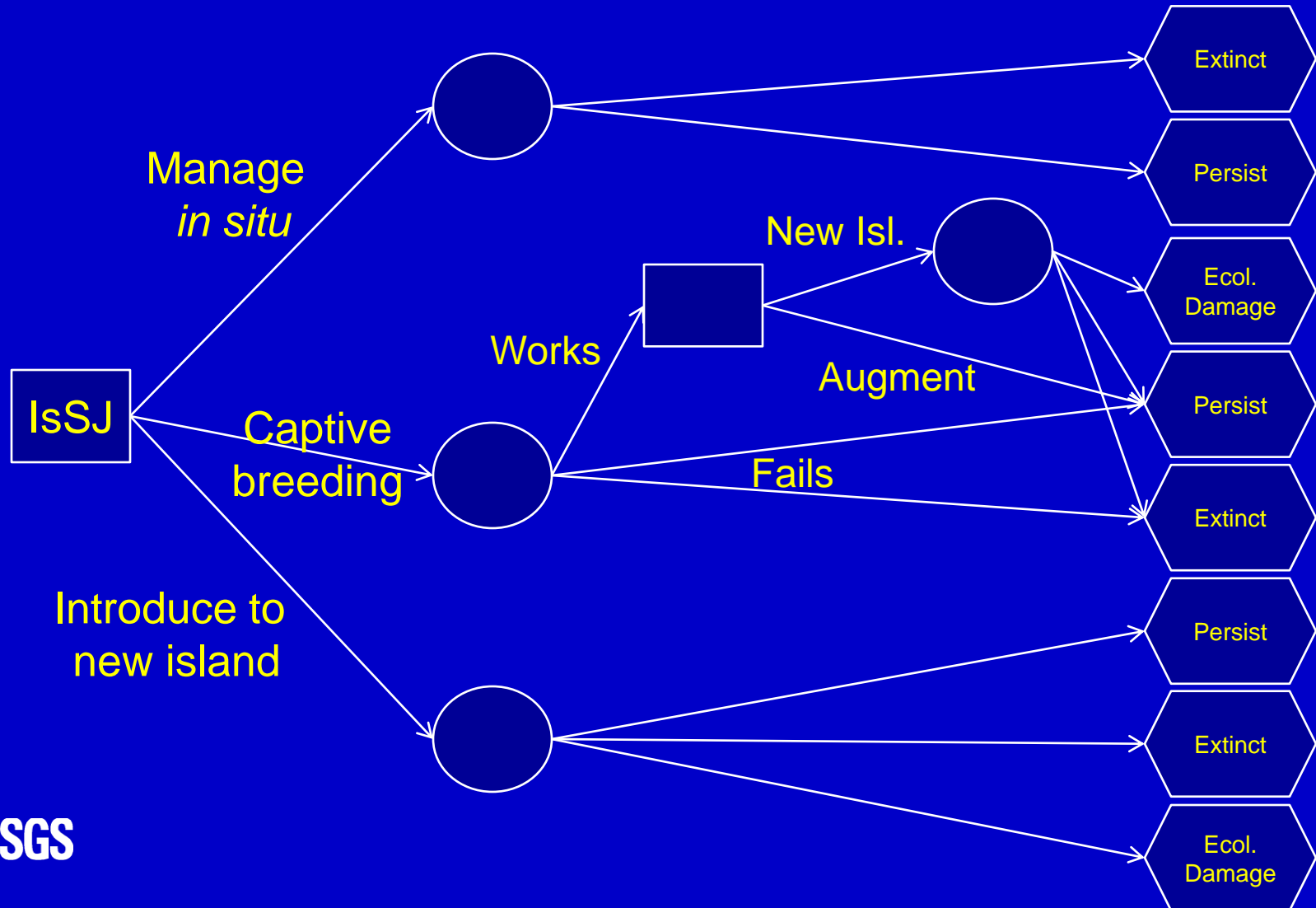


Decision Trees & Utility Theory

Michael C. Runge
USGS Patuxent Wildlife Research Center

Advanced SDM Practicum
NCTC, 12-16 March 2012

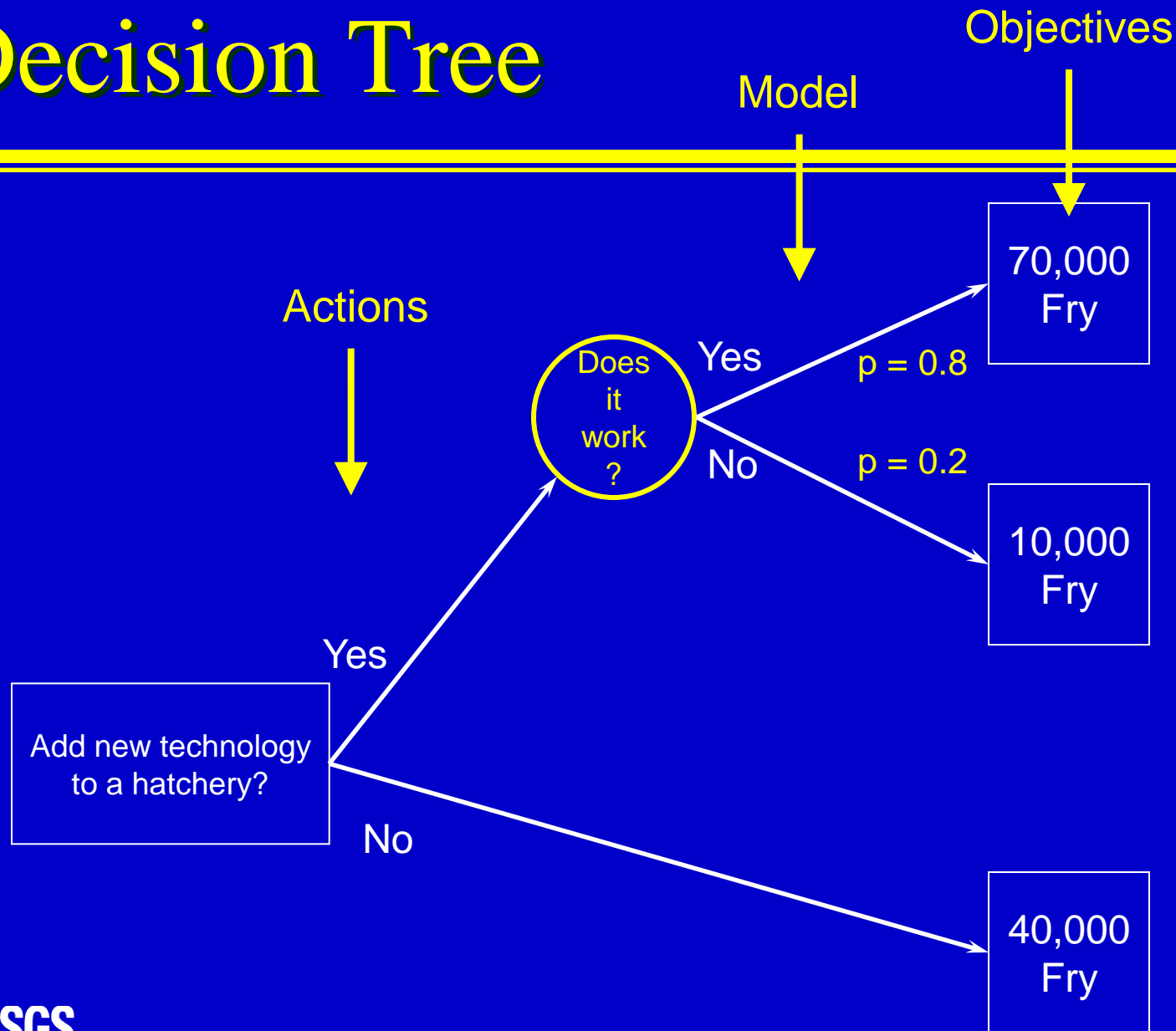
Motivation: Risk



Outline

- Decision trees
- Utility curves
- Eliciting utility curves
- Utility functions
- Multi-attribute utility
- Cognitive challenges
- A few other thoughts...

Decision Tree

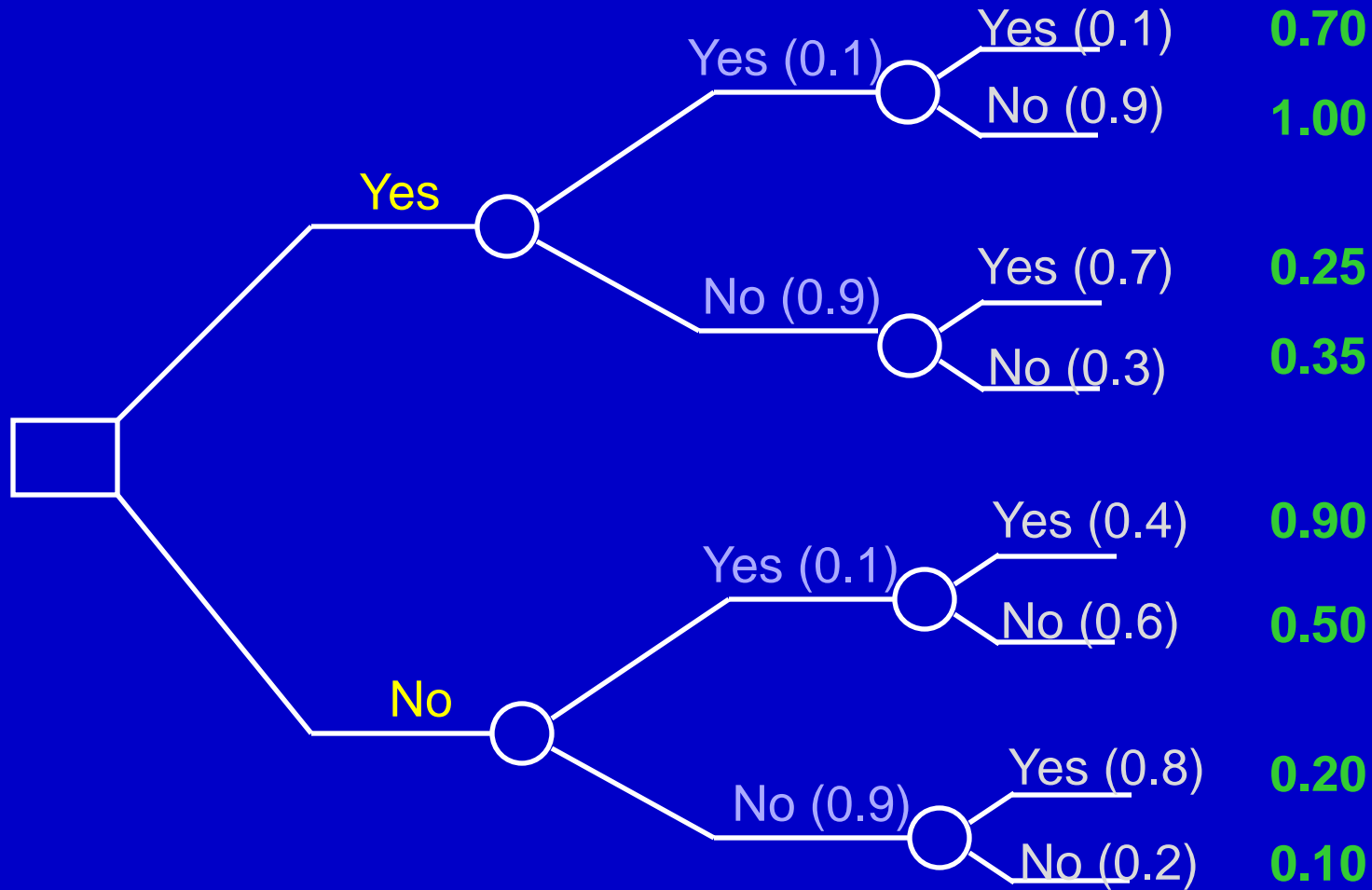


Control Burn?

Wet Year?

Wild Fire?

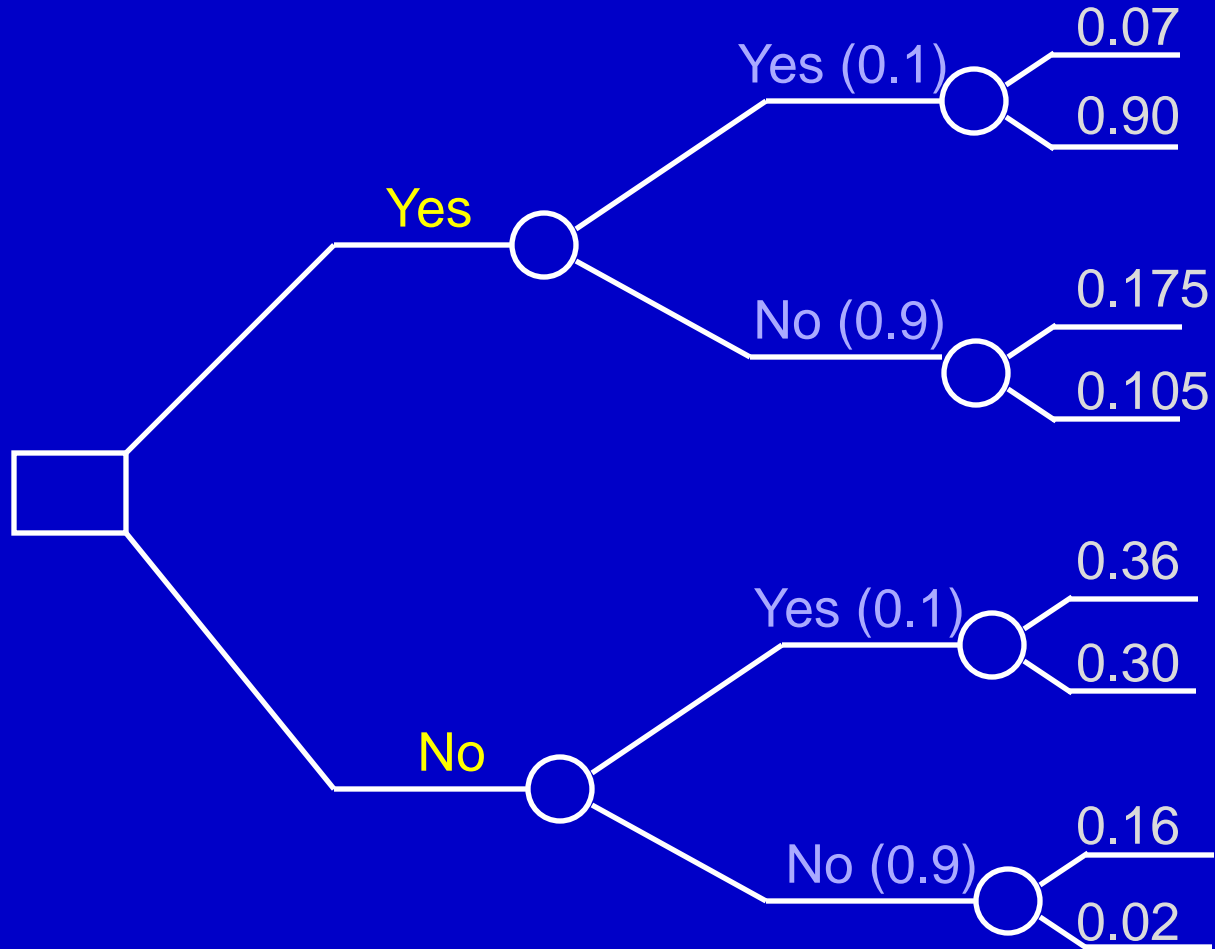
Value



Control Burn?

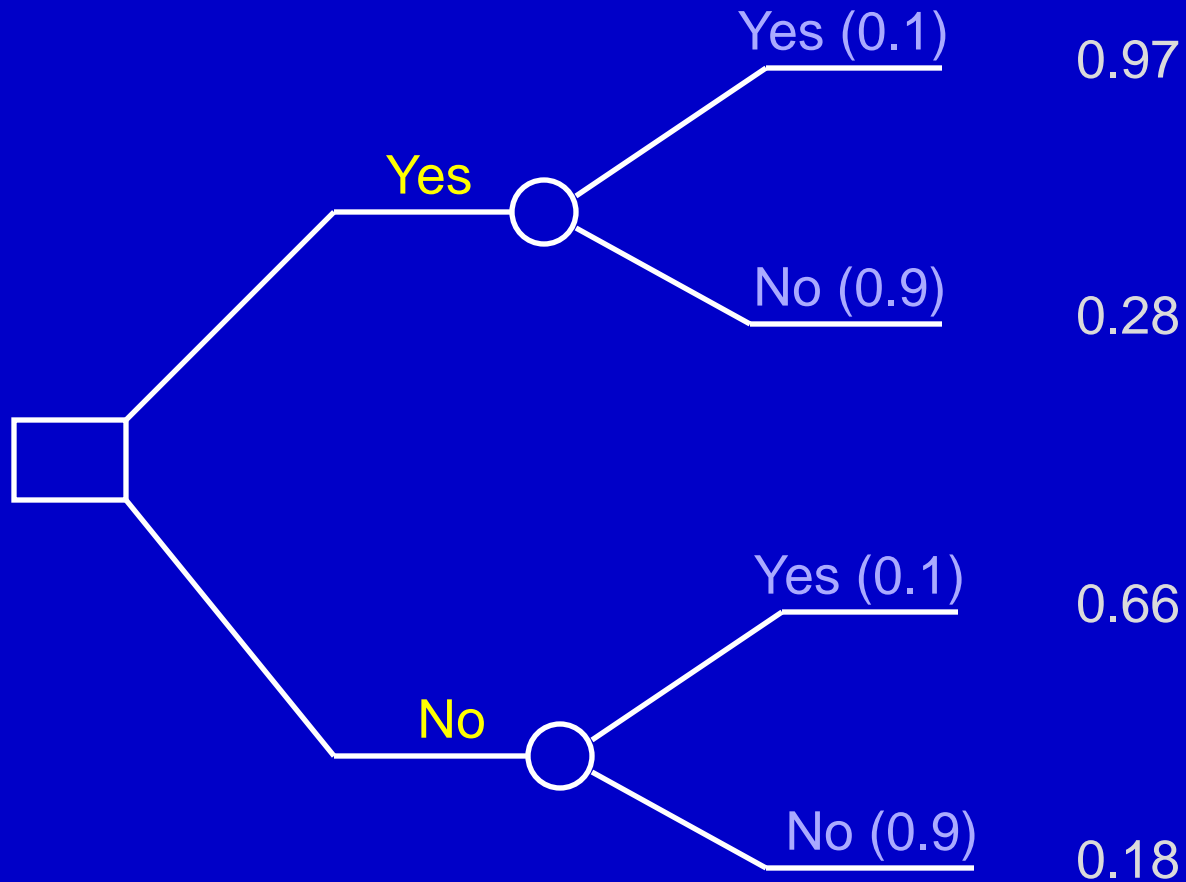
Wet Year?

Wild Fire?



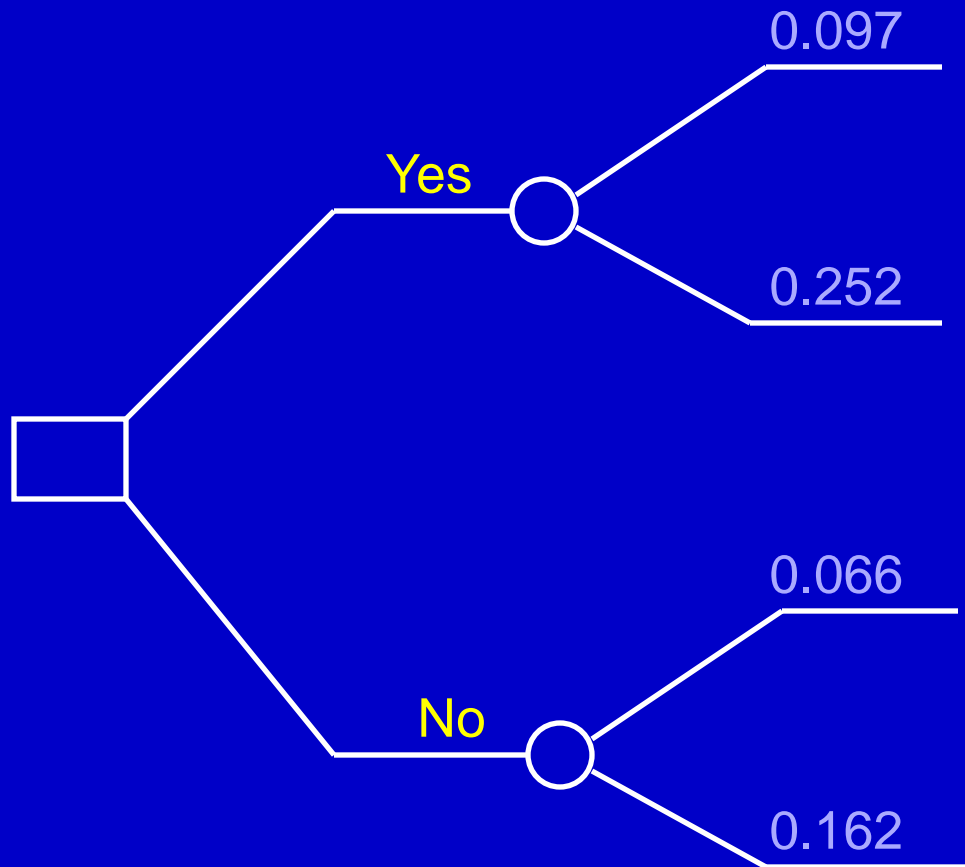
Control Burn?

Wet Year?

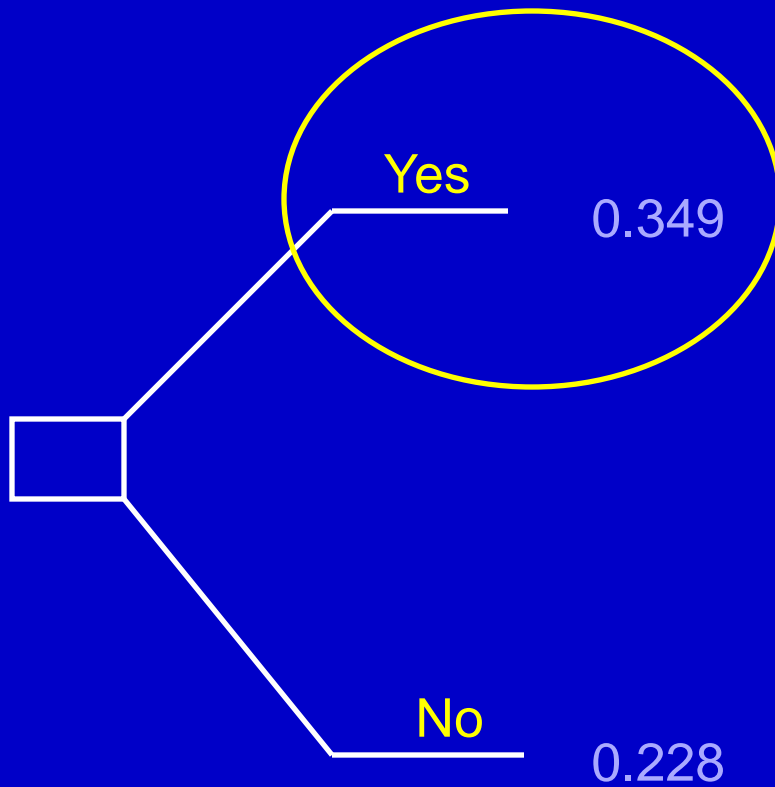


Control Burn?

Wet Year?



Control Burn?



Roll-back Method:

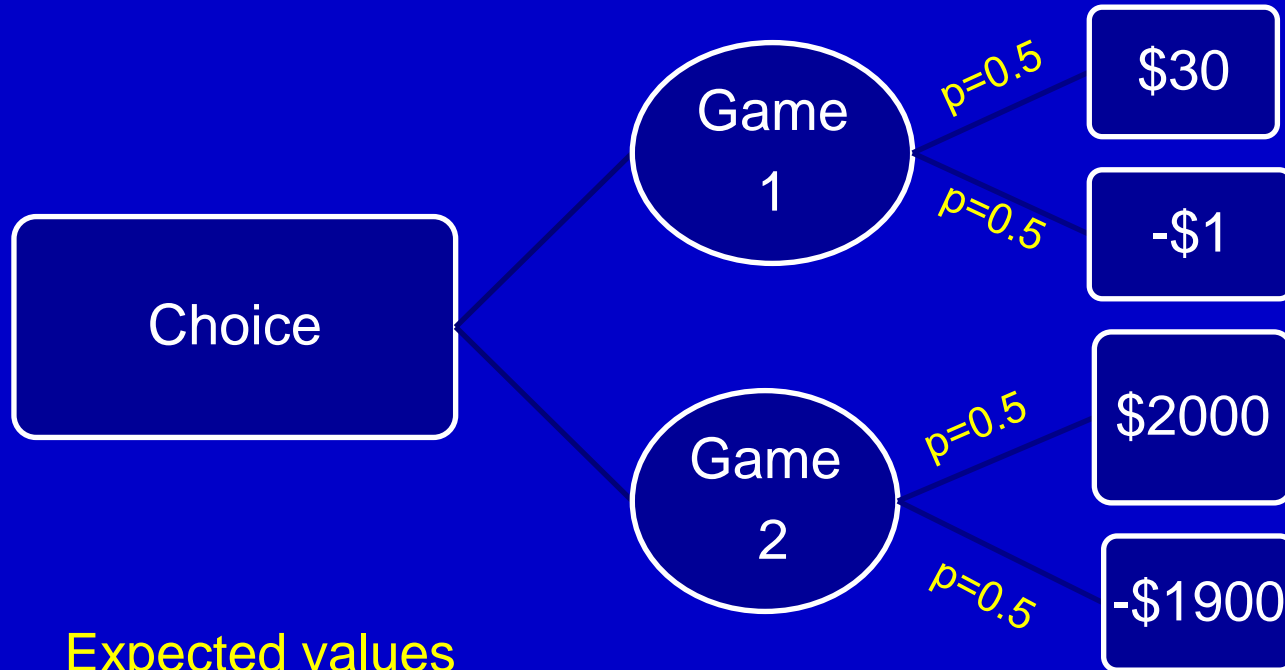
Start at right

EV at chance nodes

Best at choice nodes

Move left until done

Does EV capture values?



Expected values

Game 1: \$14.50

Game 2: \$50.00

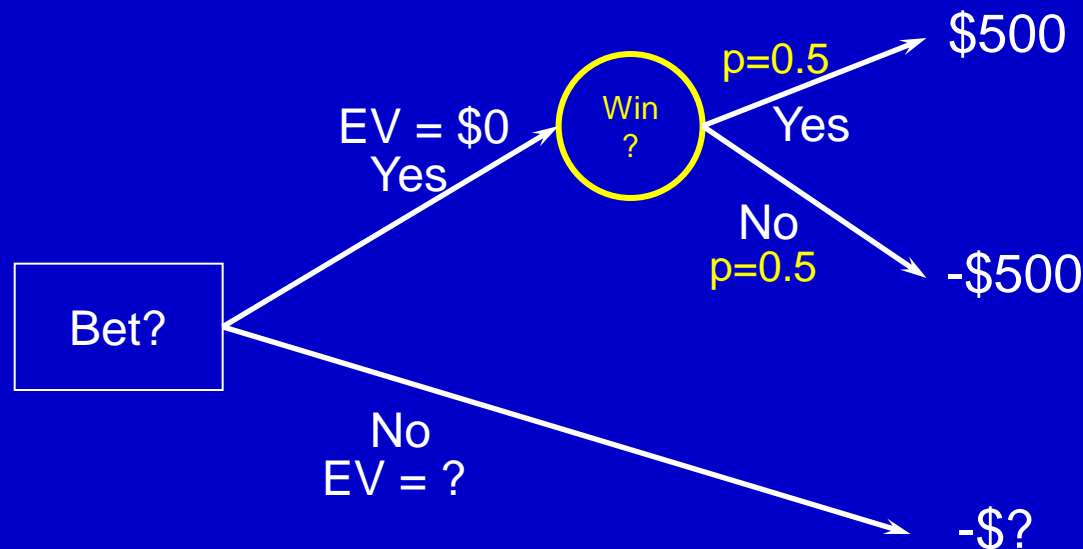
Which do you choose?

Expected Value

- The expected value criterion
 - Assumes a long-run average
 - Assumes a linear value function
 - Focuses on only a single attribute
- But maybe...
 - We make repeated decisions in our life...

Risk Attitude

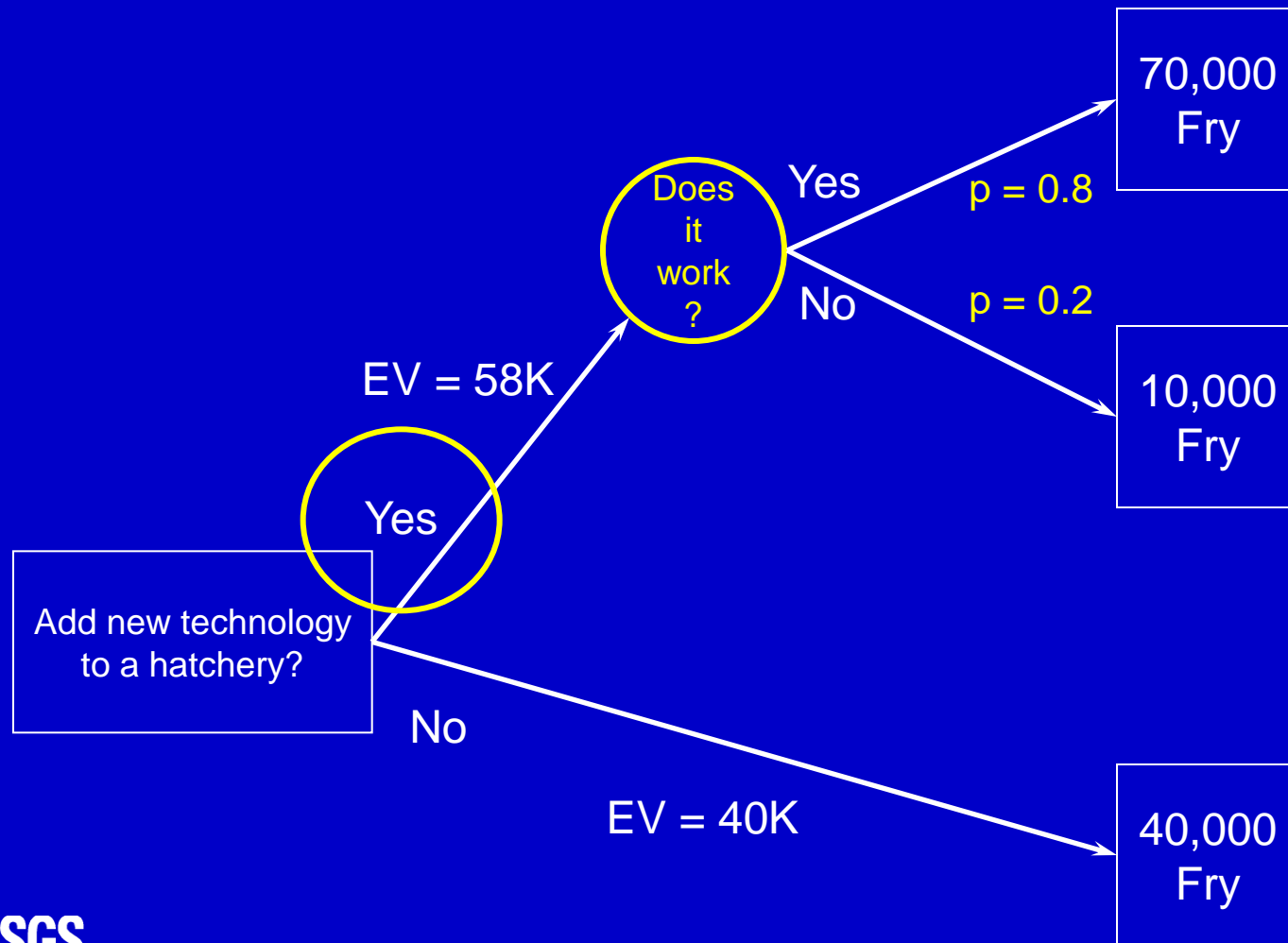
- Consider the following wager
 - Win \$500 with prob 0.5, or lose \$500 with prob 0.5
 - Would you pay to get out of this wager? How much?
 - Would you pay to get into this wager? How much?
- A classic risk decision



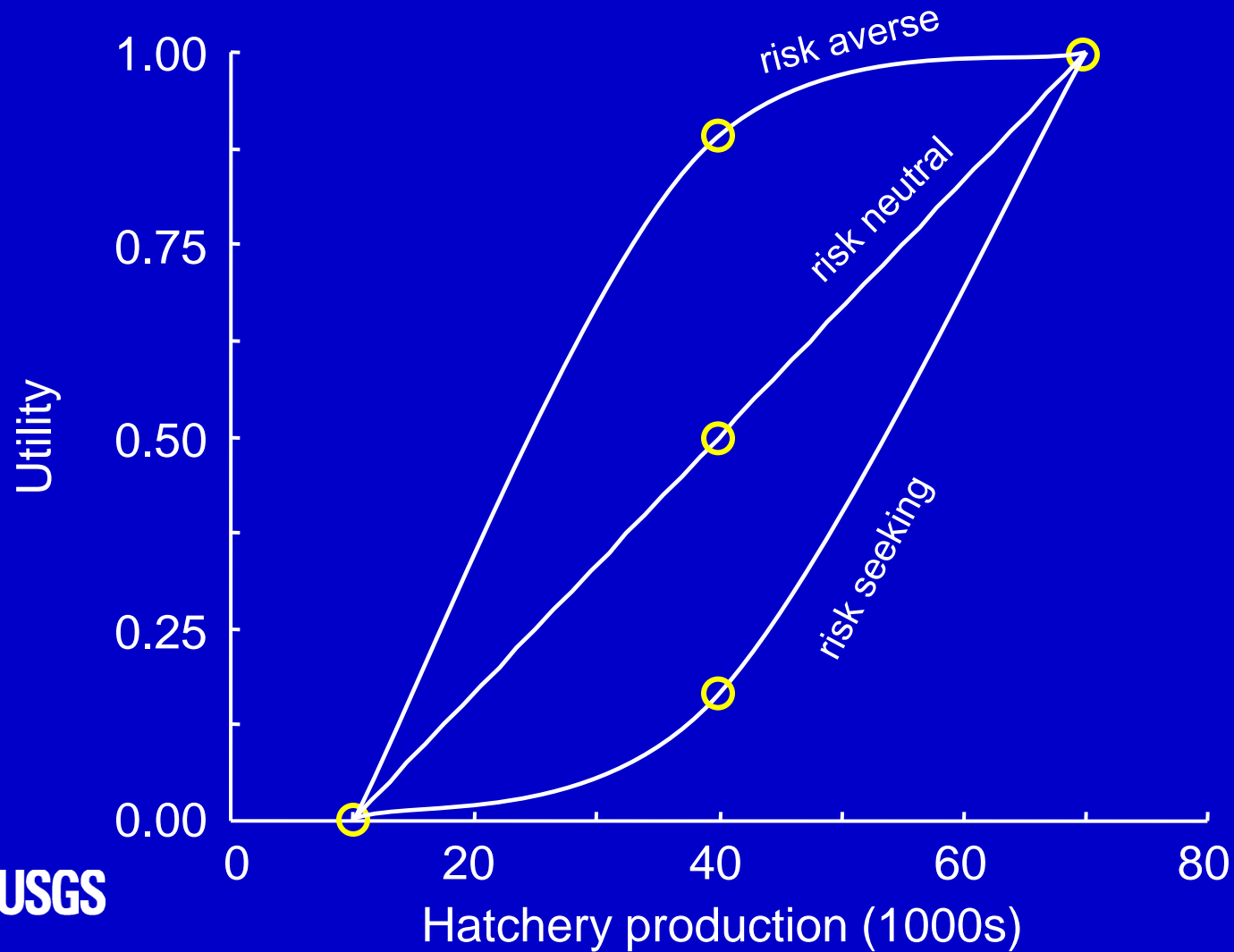
Risk Attitude

- Risk-averse
 - You would trade a gamble for a sure amount that is less than the expected value of the gamble
 - E.g., buying insurance
- Risk-seeking
 - You would trade a sure amount for a gamble that has a smaller expected value (but the chance of a larger payout)
 - E.g., buying lottery tickets

Decision Tree

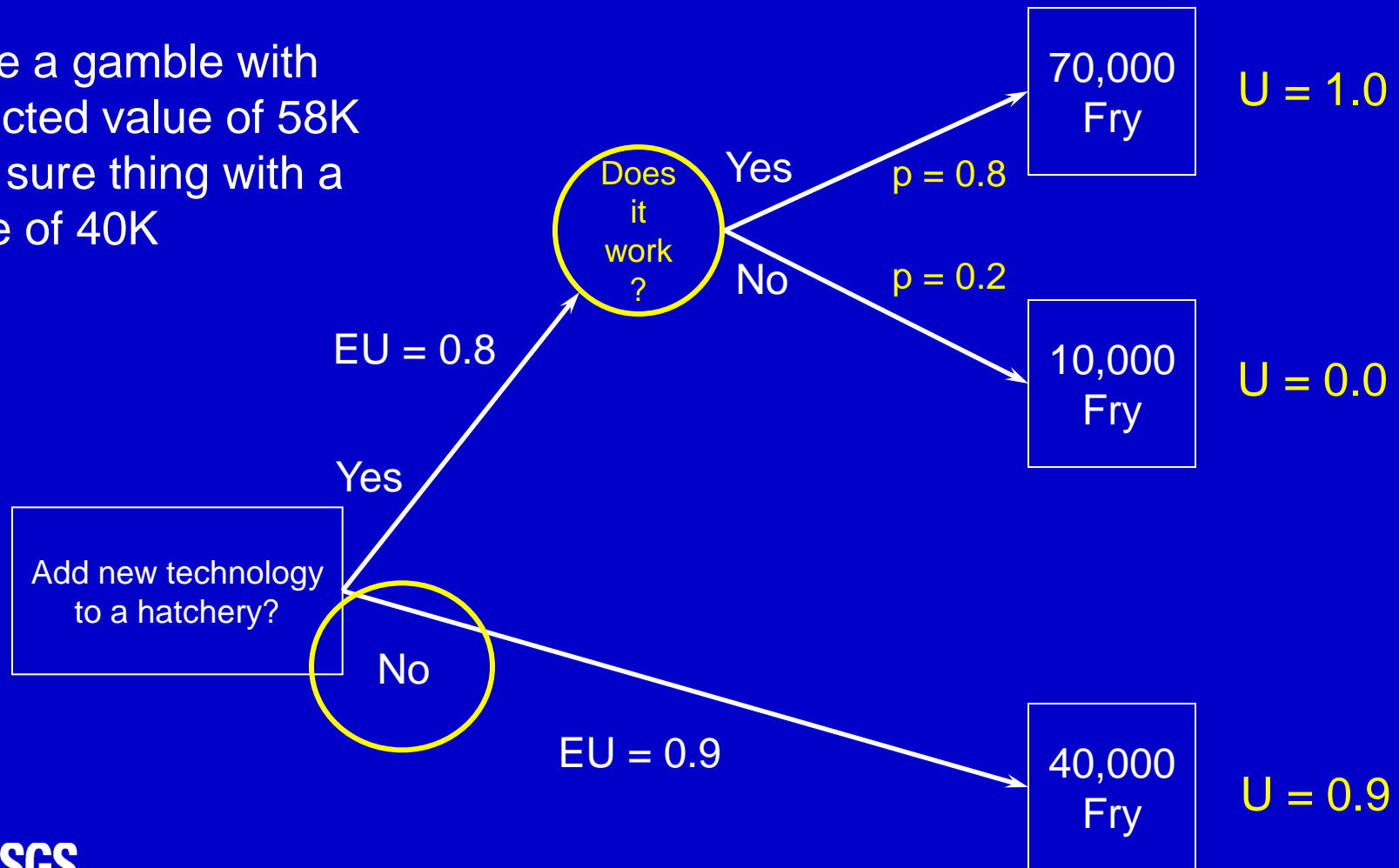


Utility



Risk-averse Utility

Trade a gamble with expected value of 58K for a sure thing with a value of 40K

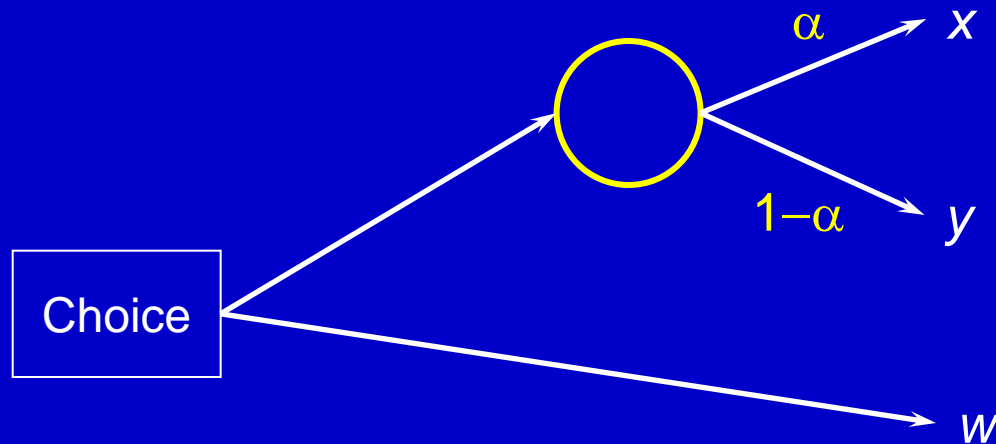


Properties of Utility Functions

- Monotonic vs. peaked
- Risk tolerance
 - Averse, neutral, seeking
 - Mixed
- Constant vs. declining aversion

Eliciting Utilities

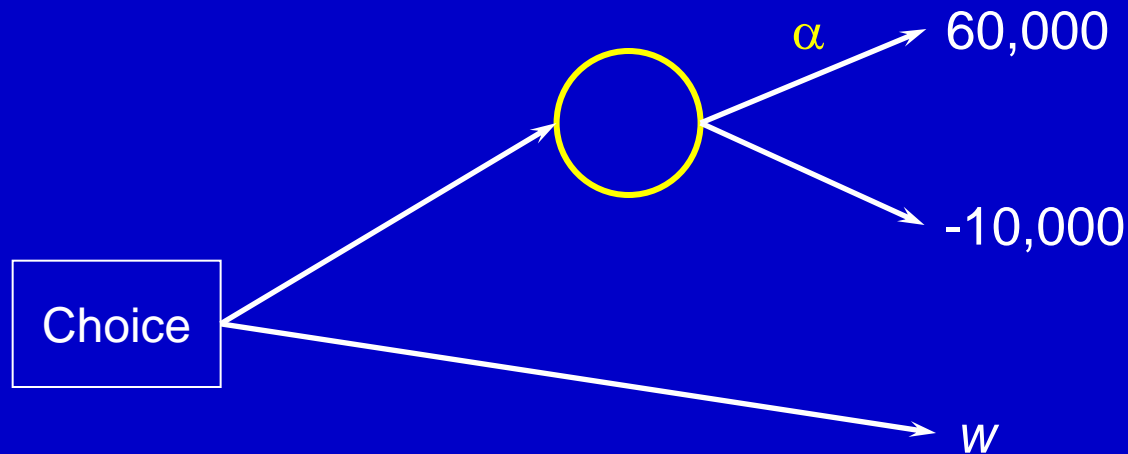
- Elicitation methods center around gamble choices
 - Notation: $[x, \alpha, y] R w$
 - The choice is between a sure return of w or gamble that returns x with probability α or y with probability $1-\alpha$
 - R is the preference relation (\succ , \prec , or \sim)
- Lottery diagram



Methods of Elicitation

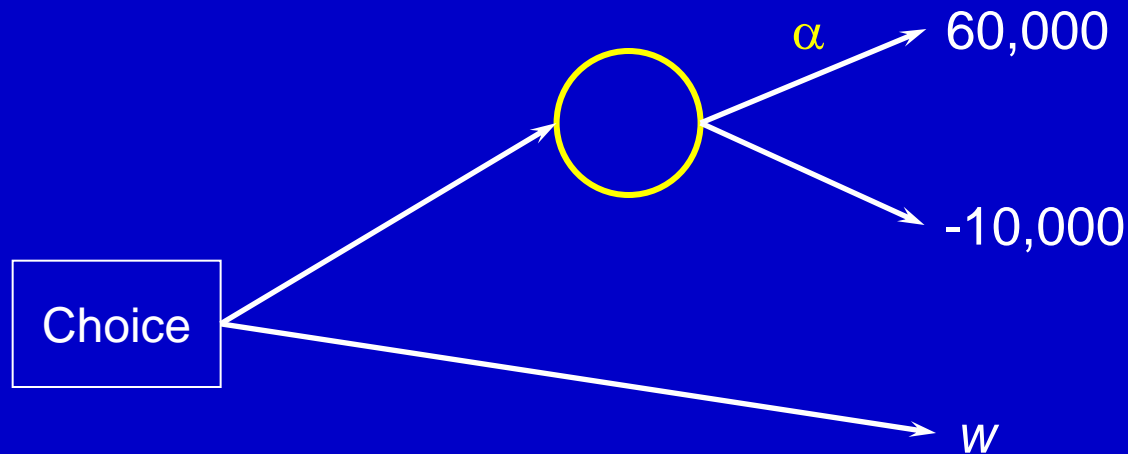
- Preference comparison
 - $[x_i, \alpha_i, y_i] R_i w_i$
- Probability equivalence
 - $[x_{n+1}, \alpha_i, x_0] \sim x_i$
- Value equivalence
- Certainty equivalence
 - $[x^*, 0.5, x_0] \sim x_1, [x_1, x_0] \sim x_2, [x^*, x_1] \sim x_3, \dots$

Probability-equivalence



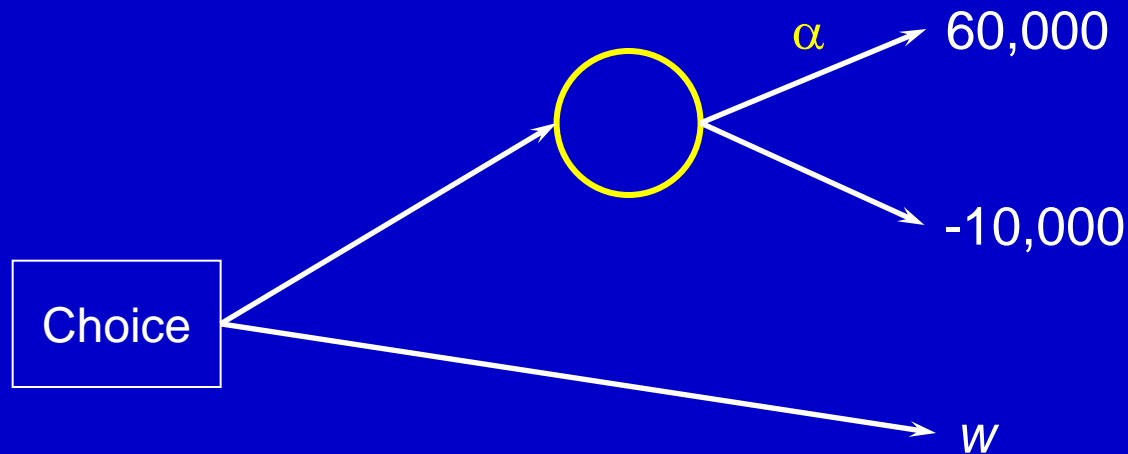
w	-10,000	0	10,000	30,000	60,000
α					
$u(w)$	0.0				1.0

Probability-equivalence



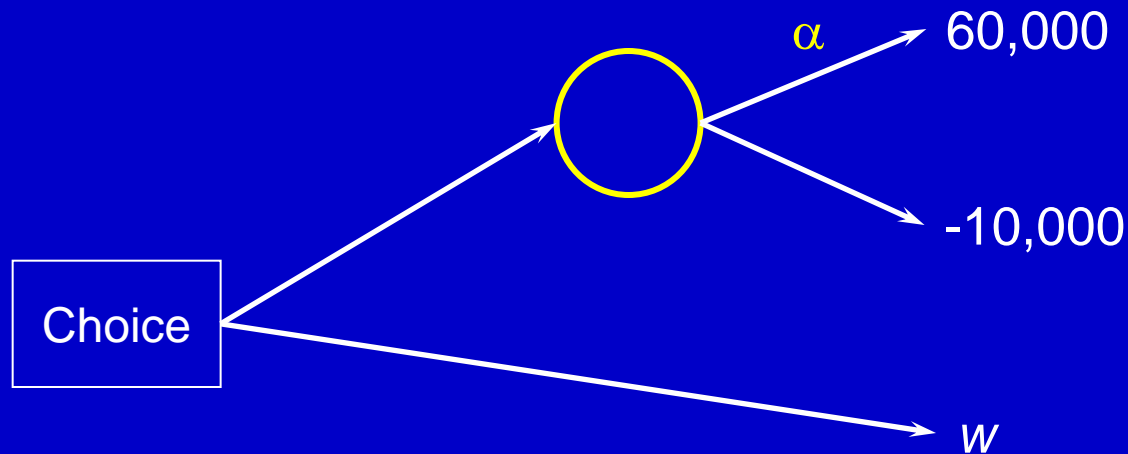
w	-10,000	0	10,000	30,000	60,000
α				0.85	
$u(w)$	0.0				1.0

Probability-equivalence



w	-10,000	0	10,000	30,000	60,000
α			0.60	0.85	
$u(w)$	0.0				1.0

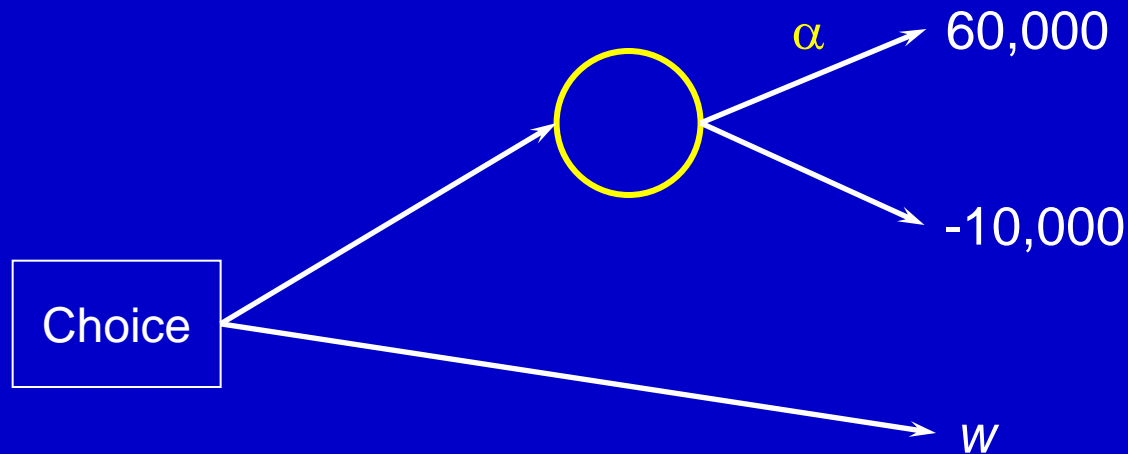
Probability-equivalence



w	-10,000	0	10,000	30,000	60,000
α		0.35	0.60	0.85	
$u(w)$	0.0				1.0

$$\begin{aligned}u(30,000) &= \alpha u(60,000) + (1 - \alpha)u(-10,000) \\ &= \alpha(1.0) + (1 - \alpha)(0.0) = \alpha\end{aligned}$$

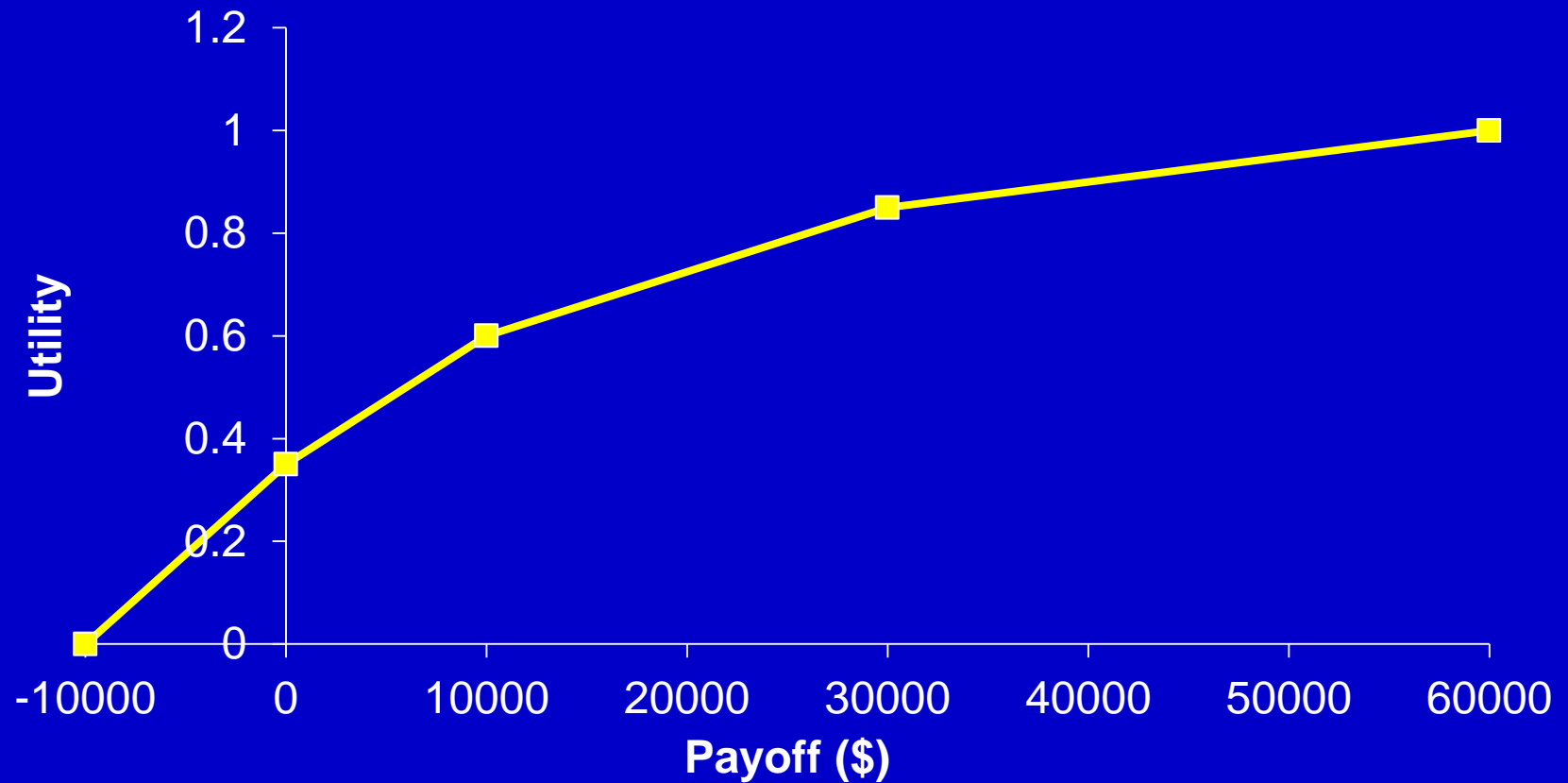
Probability-equivalence



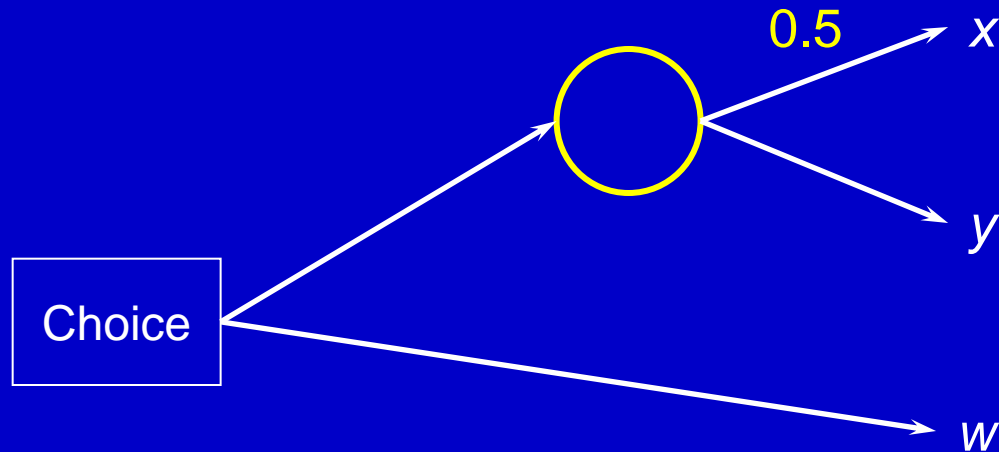
w	-10,000	0	10,000	30,000	60,000
α		0.35	0.60	0.85	
$u(w)$	0.0	0.35	0.60	0.85	1.0

$$\begin{aligned}u(30,000) &= \alpha u(60,000) + (1 - \alpha)u(-10,000) \\ &= \alpha(1.0) + (1 - \alpha)(0.0) = \alpha\end{aligned}$$

Utility Curve



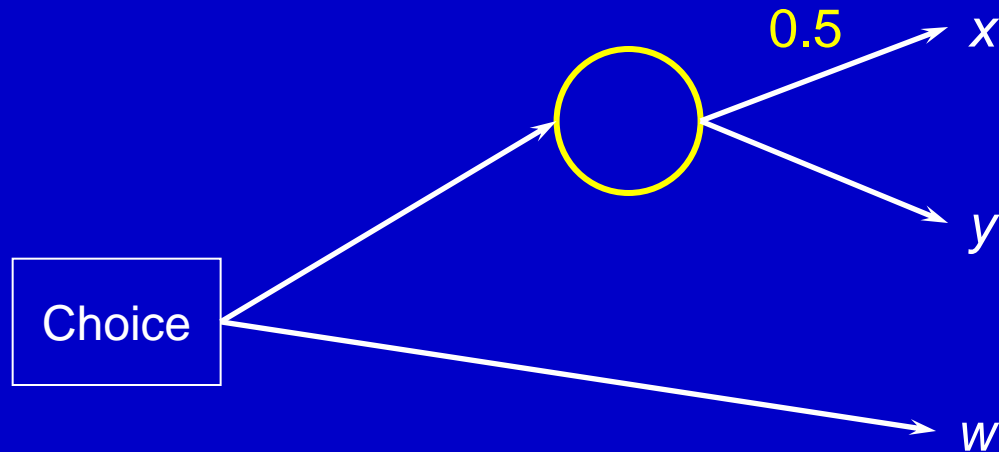
Certainty-equivalence



x	60,000						
y	-10,000						
w	w_1						
$u(w)$							

$$\begin{aligned}u(w_1) &= 0.5u(60,000) + (1 - 0.5)u(-10,000) \\ &= 0.5(1.0) + 0.5(0.0) = 0.5\end{aligned}$$

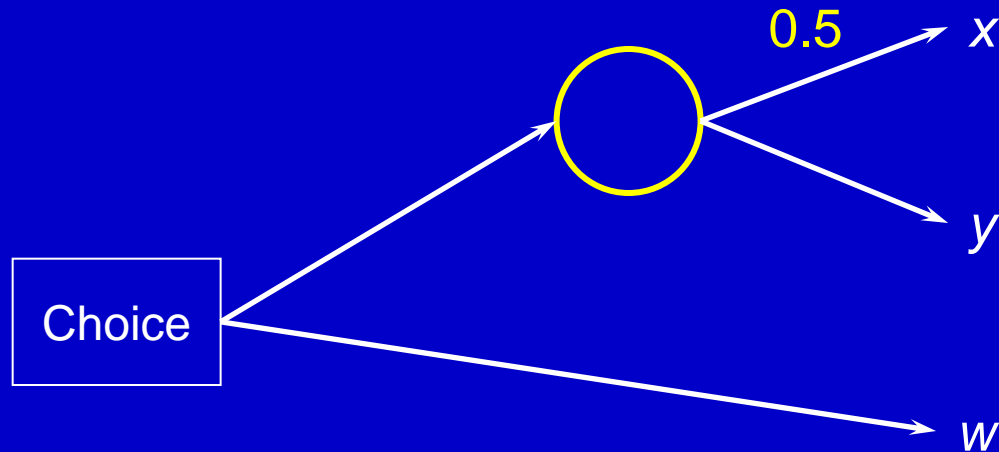
Certainty-equivalence



x	60,000	w_1					
y	-10,000	-10,000					
w	w_1	w_2					
$u(w)$	0.5						

$$\begin{aligned}
 u(w_2) &= 0.5u(w_1) + (1 - 0.5)u(-10,000) \\
 &= 0.5(0.5) + 0.5(0.0) = 0.25
 \end{aligned}$$

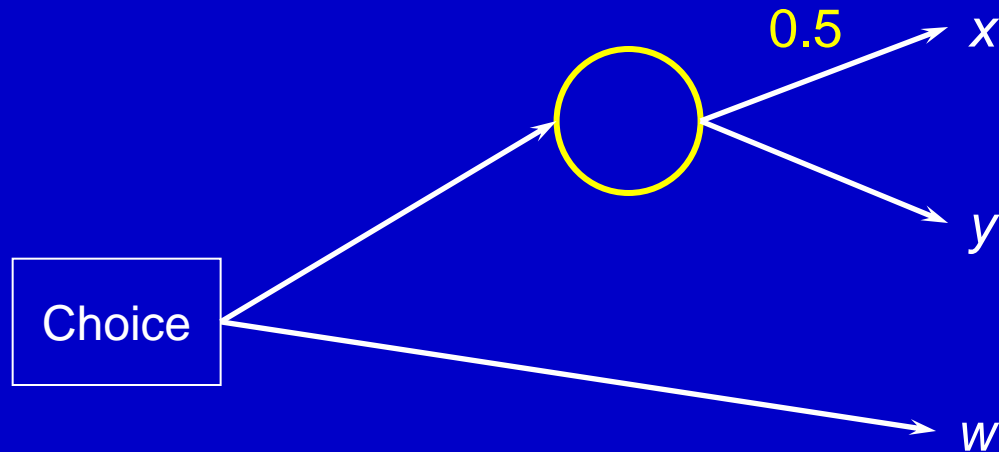
Certainty-equivalence



x	60,000	w_1	60,000	60,000	w_3	w_1	w_2
y	-10,000	-10,000	w_1	w_3	w_1	w_2	-10,000
w	w_1	w_2	w_3				
$u(w)$	0.5	0.25	0.75				

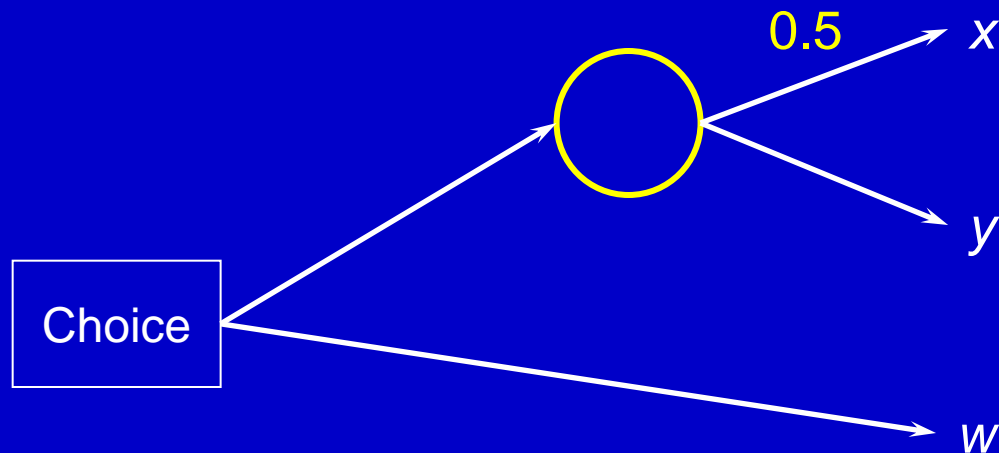
$$\begin{aligned}
 u(w_3) &= 0.5u(60,000) + (1 - 0.5)u(w_1) \\
 &= 0.5(1.0) + 0.5(0.5) = 0.75
 \end{aligned}$$

Certainty-equivalence



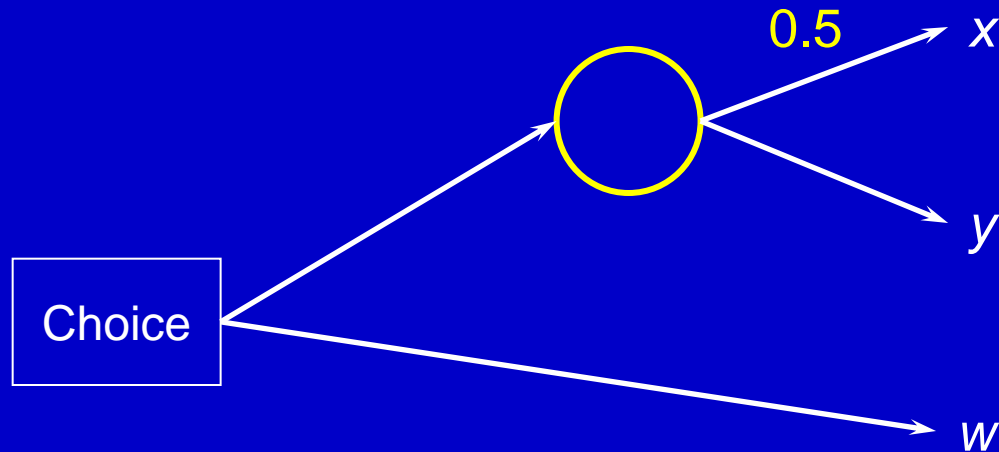
x	60,000	w_1	60,000	60,000	w_3	w_1	w_2
y	-10,000	-10,000	w_1	w_3	w_1	w_2	-10,000
w	w_1	w_2	w_3				
$u(w)$	0.5	0.25	0.75	0.875	0.625	0.375	0.125

Certainty-equivalence



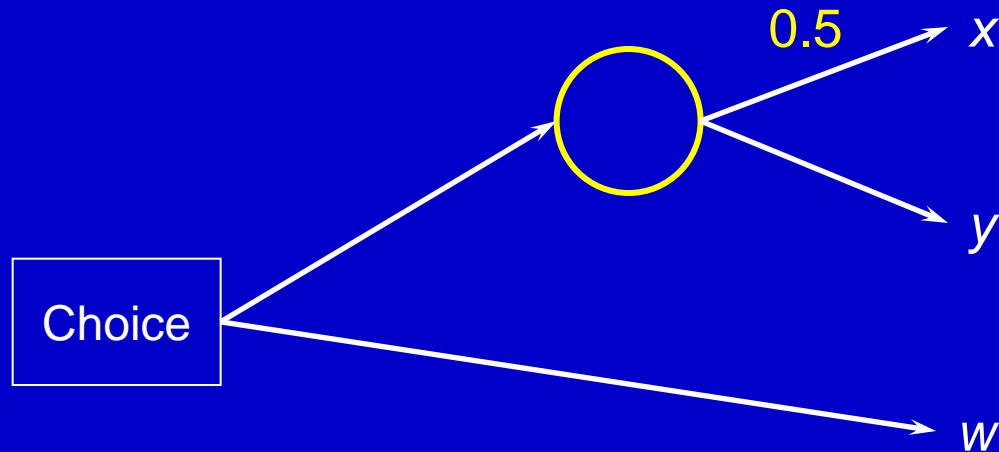
x	60,000	8,000	60,000	60,000	w_3	8,000	w_2
y	-10,000	-10,000	8,000	w_3	8,000	w_2	-10,000
w	8,000	w_2	w_3				
$u(w)$	0.5	0.25	0.75	0.875	0.625	0.375	0.125

Certainty-equivalence



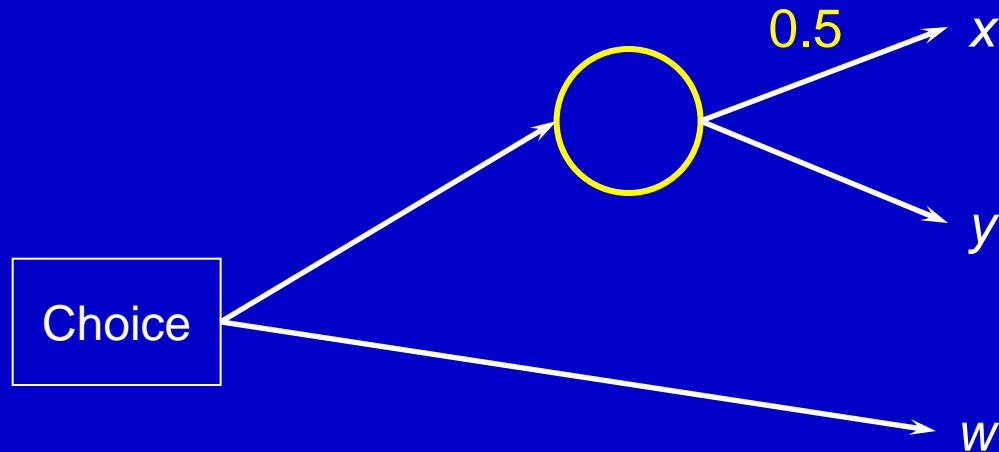
x	60,000	8,000	60,000	60,000	w_3	8,000	2,000
y	-10,000	-10,000	8,000	w_3	8,000	2,000	-10,000
w	8,000	-2,000	w_3				
$u(w)$	0.5	0.25	0.75	0.875	0.625	0.375	0.125

Certainty-equivalence



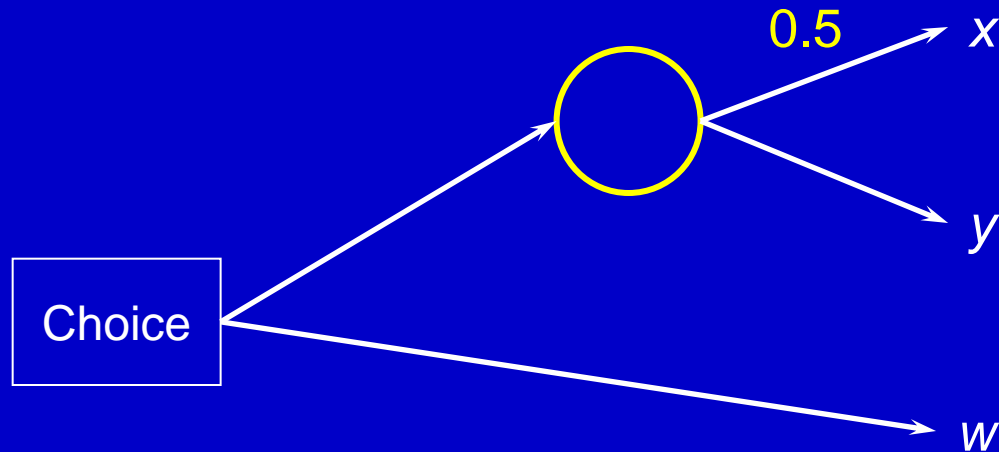
x	60,000	8,000	60,000	60,000	20,000	8,000	2,000
y	-10,000	-10,000	8,000	20,000	8,000	2,000	-10,000
w	8,000	-2,000	20,000				
$u(w)$	0.5	0.25	0.75	0.875	0.625	0.375	0.125

Certainty-equivalence



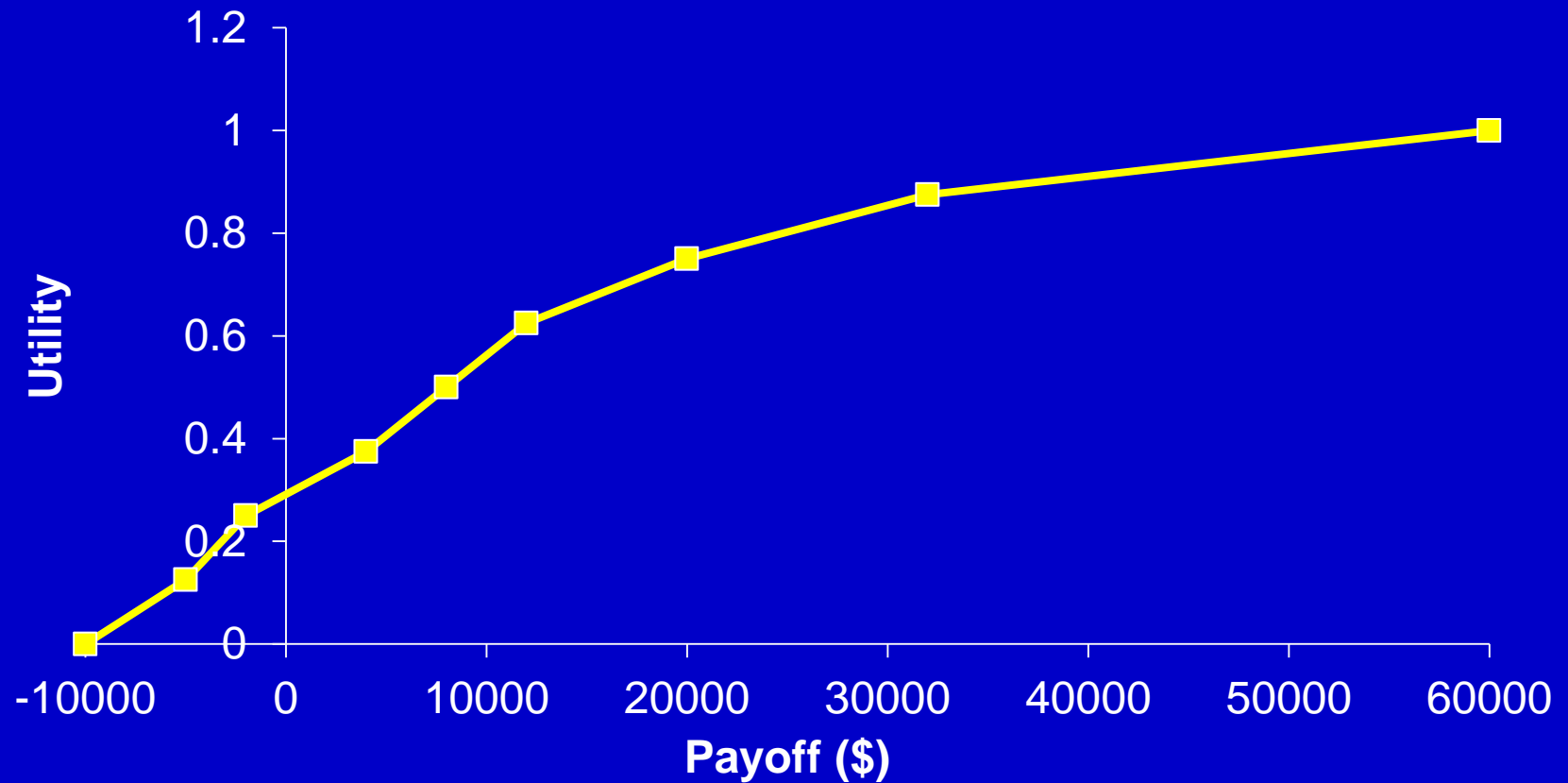
x	60,000	8,000	60,000	60,000	20,000	8,000	2,000
y	-10,000	-10,000	8,000	20,000	8,000	2,000	-10,000
w	8,000	-2,000	20,000	32,000			
$u(w)$	0.5	0.25	0.75	0.875	0.625	0.375	0.125

Certainty-equivalence



x	60,000	8,000	60,000	60,000	20,000	8,000	2,000
y	-10,000	-10,000	8,000	20,000	8,000	2,000	-10,000
w	8,000	-2,000	20,000	32,000	12,000	4,000	-5,000
$u(w)$	0.5	0.25	0.75	0.875	0.625	0.375	0.125

Utility Curve



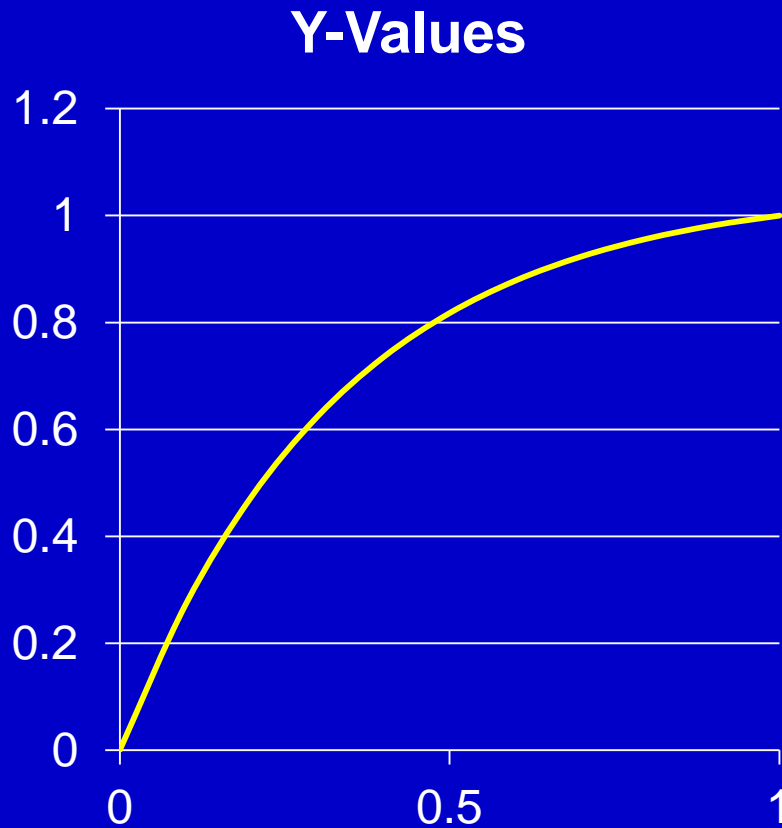
Methods of Elicitation

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- Certainty equivalence
 - $[x^*, 0.5, x_0] \sim x_1, [x_1, x_0] \sim x_2, [x^*, x_1] \sim x_3, \dots$

Utility Functions

- There are functions that describe smooth utility curves
 - Compact expressions
 - These are often easier to elicit than a lot of individual points
- Common
 - Linear
 - Exponential
 - Logarithmic

Exponential Utility



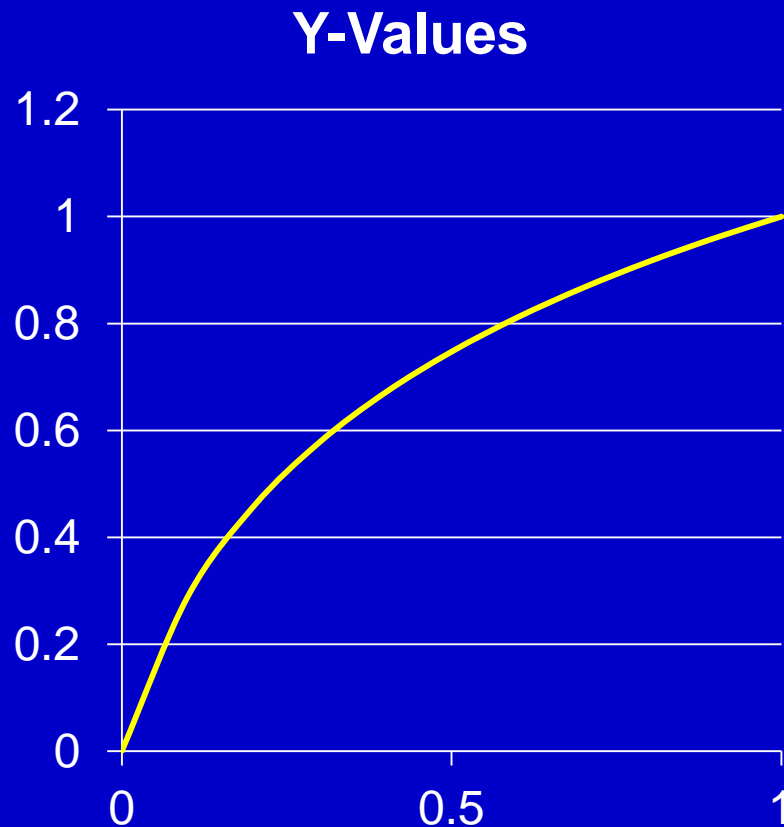
- Kernel

- e^{-cx}

- Risk attitude

- $c > 0$, risk averse
 - $c < 0$, risk seeking
 - constant

Logarithmic Utility



- Kernel
 - $\log(x + b)$
 - $x > b$
- Risk attitude
 - risk averse
 - declining

Scaling

- Utility functions can be scaled to the interval $\{0,1\}$
 - Linear transformation
- $$u(x) = \frac{k(x) - k(x_0)}{k(x_1) - k(x_0)}$$

Multi-attribute Utility

- What if there is more than one objective?
- Most commonly
 - Assume mutual utility independence
 - Develop utilities separately
 - Combine into single expression
- Goodwin & Wright (2004:123ff)

Cognitive Challenges

- Lotteries are imaginary
- Subtleties of elicitation
 - Gift, purchase, sale, transfer
- Strength of preference for sure outcomes vs. attitudes toward risk

Recommendations

- Pre-analysis preparation phase
 - Motivate decision maker to think carefully about responses
- Use more than one assessment procedure
- Phrase utility questions in terms closely related to original problem

A few more thoughts...

- Value vs. utility
- “Unknown unknowns”